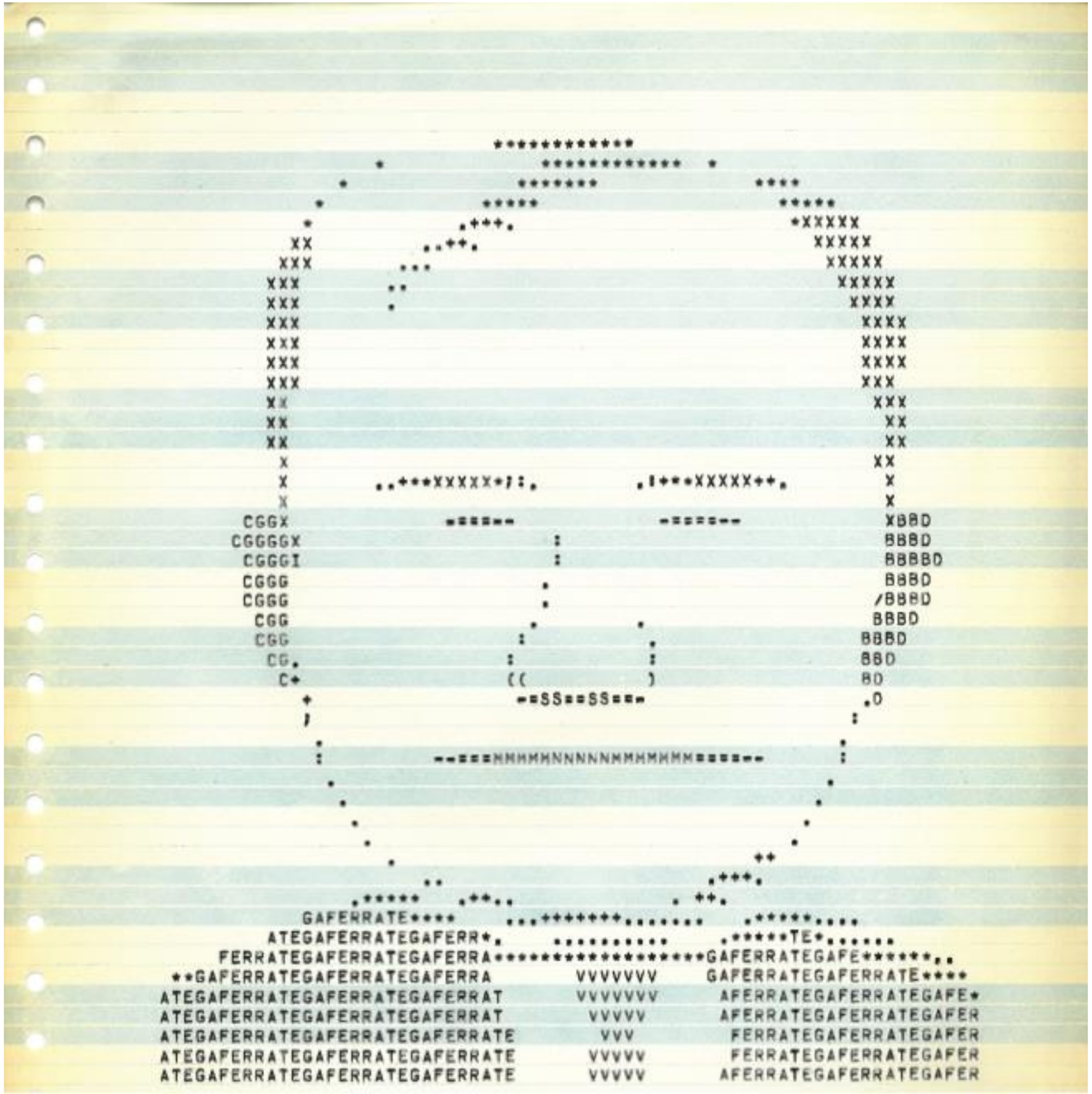
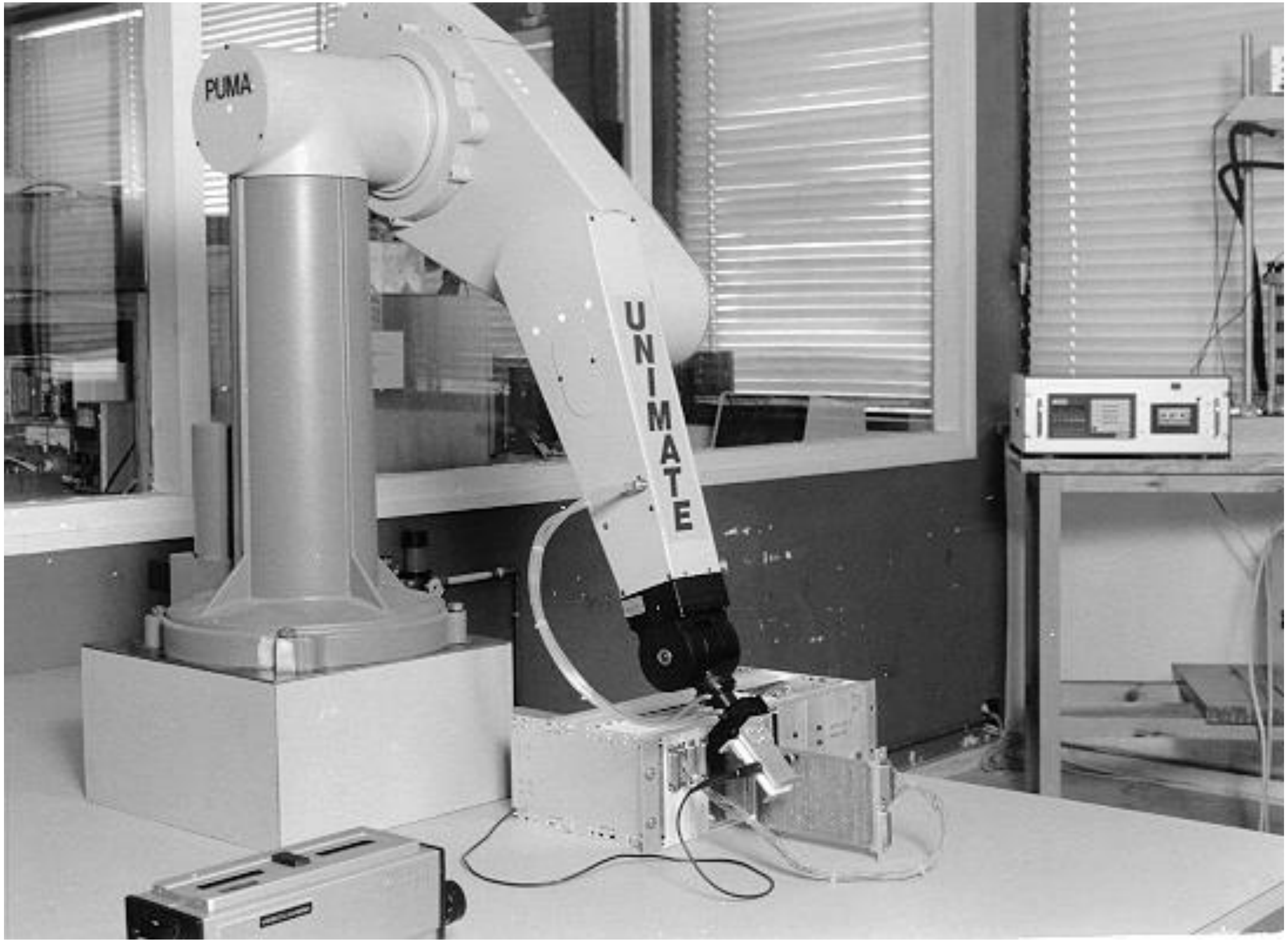
GABRIEL FERRATÉ

Mestre de professors i investigadors

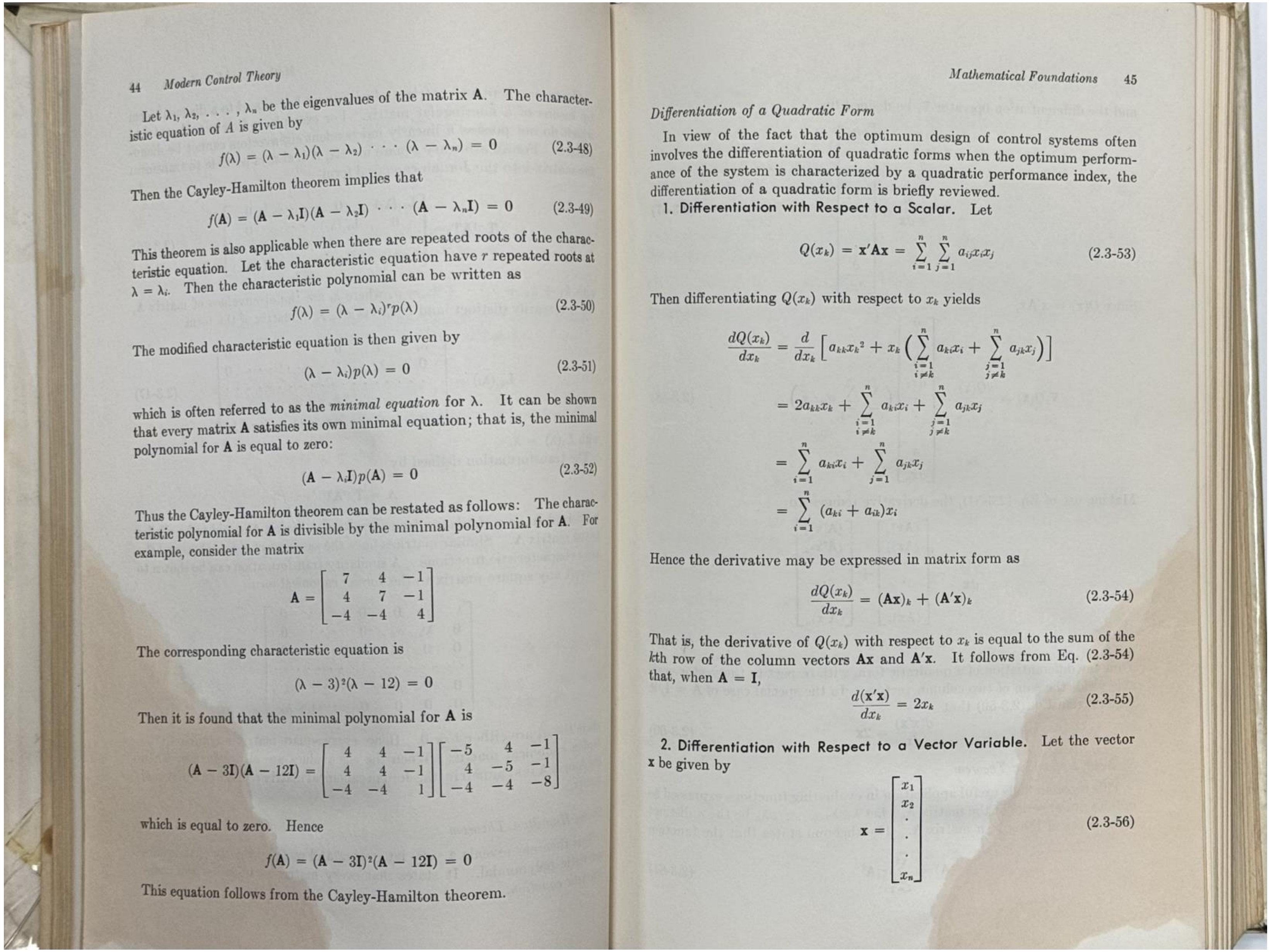


Caricatura de Ferraté realitzada amb el computador de l'IIC el 1972

L'Institut de Cibernètica a la planta 2 de l'edifici H de l’ETSEIB



Primer robot industrial de l’IC



44 *Modern Control Theory*

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of the matrix **A**. The characteristic equation of A is given by

$$f(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n) = 0 \qquad (2.3\text{-}48)$$

Then the Cayley-Hamilton theorem implies that

$$f(\mathbf{A}) = (\mathbf{A} - \lambda_1\mathbf{I})(\mathbf{A} - \lambda_2\mathbf{I}) \cdots (\mathbf{A} - \lambda_n\mathbf{I}) = 0 \qquad (2.3\text{-}49)$$

This theorem is also applicable when there are repeated roots of the characteristic equation. Let the characteristic equation have r repeated roots at $\lambda = \lambda_i$. Then the characteristic polynomial can be written as

$$f(\lambda) = (\lambda - \lambda_i)^r p(\lambda) \qquad (2.3\text{-}50)$$

The modified characteristic equation is then given by

$$(\lambda - \lambda_i)p(\lambda) = 0 \qquad (2.3\text{-}51)$$

which is often referred to as the *minimal equation* for λ. It can be shown that every matrix **A** satisfies its own minimal equation; that is, the minimal polynomial for **A** is equal to zero:

$$(\mathbf{A} - \lambda_i\mathbf{I})p(\mathbf{A}) = 0 \qquad (2.3\text{-}52)$$

Thus the Cayley-Hamilton theorem can be restated as follows: The characteristic polynomial for **A** is divisible by the minimal polynomial for **A**. For example, consider the matrix

$$\mathbf{A} = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$

The corresponding characteristic equation is

$$(\lambda - 3)^2(\lambda - 12) = 0$$

Then it is found that the minimal polynomial for **A** is

$$(\mathbf{A} - 3\mathbf{I})(\mathbf{A} - 12\mathbf{I}) = \begin{bmatrix} 4 & 4 & -1 \\ 4 & 4 & -1 \\ -4 & -4 & 1 \end{bmatrix} \begin{bmatrix} -5 & 4 & -1 \\ 4 & -5 & -1 \\ -4 & -4 & -8 \end{bmatrix}$$

which is equal to zero. Hence

$$f(\mathbf{A}) = (\mathbf{A} - 3\mathbf{I})^2(\mathbf{A} - 12\mathbf{I}) = 0$$

This equation follows from the Cayley-Hamilton theorem.

Mathematical Foundations 45

Differentiation of a Quadratic Form

In view of the fact that the optimum design of control systems often involves the differentiation of quadratic forms when the optimum performance of the system is characterized by a quadratic performance index, the differentiation of a quadratic form is briefly reviewed.

1. Differentiation with Respect to a Scalar. Let

$$Q(x_k) = \mathbf{x}'\mathbf{A}\mathbf{x} = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_ix_j \qquad (2.3\text{-}53)$$

Then differentiating $Q(x_k)$ with respect to x_k yields

$$\begin{aligned} \frac{dQ(x_k)}{dx_k} &= \frac{d}{dx_k}\left[a_{kk}x_k^2 + x_k\left(\sum_{\substack{i=1\\i\neq k}}^{n} a_{ki}x_i + \sum_{\substack{j=1\\j\neq k}}^{n} a_{jk}x_j\right)\right] \\ &= 2a_{kk}x_k + \sum_{\substack{i=1\\i\neq k}}^{n} a_{ki}x_i + \sum_{\substack{j=1\\j\neq k}}^{n} a_{jk}x_j \\ &= \sum_{i=1}^{n} a_{ki}x_i + \sum_{j=1}^{n} a_{jk}x_j \\ &= \sum_{i=1}^{n} (a_{ki} + a_{ik})x_i \end{aligned}$$

Hence the derivative may be expressed in matrix form as

$$\frac{dQ(x_k)}{dx_k} = (\mathbf{A}\mathbf{x})_k + (\mathbf{A}'\mathbf{x})_k \qquad (2.3\text{-}54)$$

That is, the derivative of $Q(x_k)$ with respect to x_k is equal to the sum of the kth row of the column vectors **Ax** and **A′x**. It follows from Eq. (2.3-54) that, when $\mathbf{A} = \mathbf{I}$,

$$\frac{d(\mathbf{x}'\mathbf{x})}{dx_k} = 2x_k \qquad (2.3\text{-}55)$$

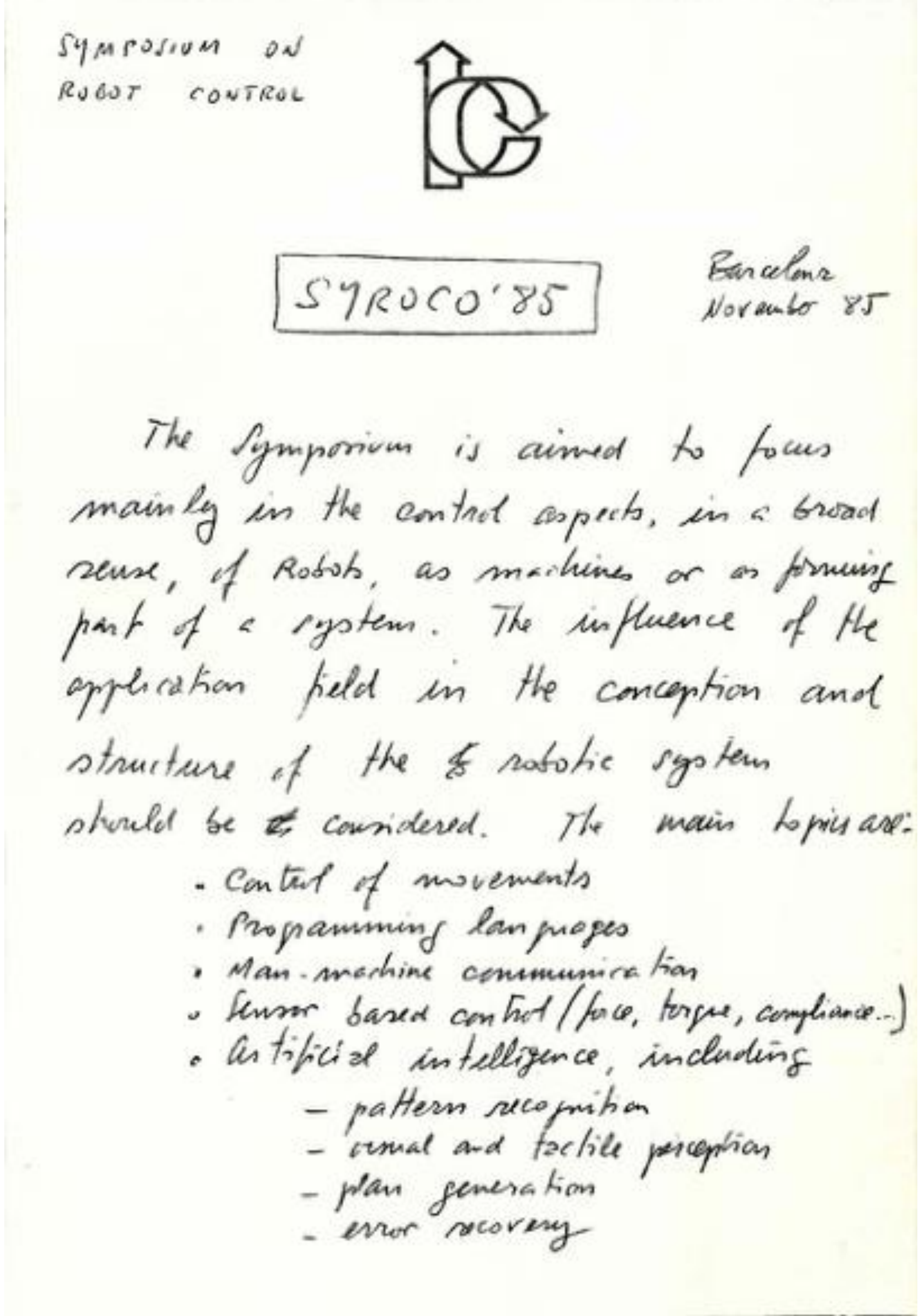
2. Differentiation with Respect to a Vector Variable. Let the vector $\mathbf{x}$ be given by

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \qquad (2.3\text{-}56)$$

El llibre
"Modern Control Theory"
enaltit pel vi

Gabriel Ferraté treballant amb el computador híbrid de l’IC

IFAC WC 1999: representació espanyola encapçalada per Ferraté
al Gran Palau del Poble de Beijing



SYMPOSIUM ON
ROBOT CONTROL

SYROCO'85

Barcelona
Novembre 85

The Symposium is aimed to focus mainly in the control aspects, in a broad sense, of Robots, as machines or as forming part of a system. The influence of the application field in the conception and structure of the robotic system should be considered. The main topics are:

- Control of movements
- Programming languages
- Man-machine communication
- Sensor based control (force, torque, compliance...)
- Artificial intelligence, including
  - pattern recognition
  - visual and tactile perception
  - plan generation
  - error recovery

Manuscrit de Ferraté de la proposta inicial del SYROCO

Taula presidencial de la inauguració de l'IFAC WC 2002
al Liceu de Barcelona

RECONEIXEMENTS de la activitat en el món científic i universitari de Gabriel Ferraté

Membre numerari de l'Institut d'Estudis Catalans
Membre numerari de l'Acadèmia de Ciències i Arts de Barcelona
Membre numerari de la Reial Acadèmia de Medicina de Catalunya
Acadèmic de número de la Academia de Ingeniería ,
Membre de l'Acadèmia de Ciències i Arts Europea
Membre del Patronat de la Fundació Enciclopèdia Catalana
Gran Cruz de Alfonso X el Sabio'
Ordre des Palmes Acadèmiques, en grau d'oficial, de l'Estat francès

Medalla Narcís Monturiol de la Generalitat de Catalunya al mèrit científic i tecnològic
Medalla Konstantin E. Ciolkovskij de la Federació Cosmonàutica de la URSS
Medalla Barcelona '92, de l'Ajuntament de Barcelona.

Doctor Honoris Causa per la Universidad Politécnica de Madrid (1995)
Creu de Sant Jordi de la Generalitat de Catalunya (1996)
Medalla d'or de la Ciutat de Barcelona al mèrit científi de l'Ajuntament de Barcelona (1998)
Medalla d'honor al foment de la invenció Fundación García Cabrerizo (1998)